Partially Dynamic Concurrent Update of Distributed Shortest Paths

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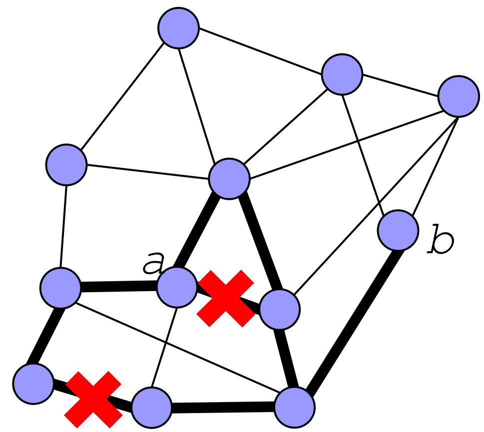
Purpose

Update shortest paths in a graph representing a distributed asynchronous system (e.g. the Internet) when edge changes occur

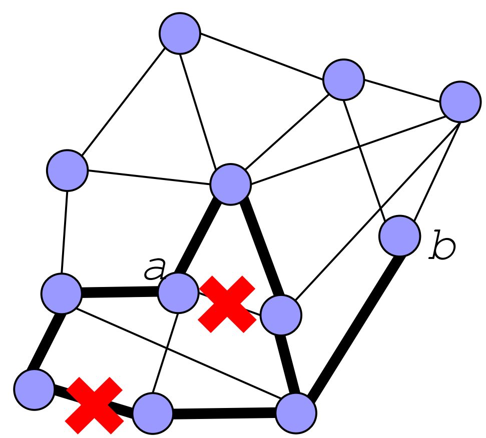
- Admitted edge changes:
  - weight increase/delete
  - weight decrease/insert
- The changes can occur in an unpredictable way (*concurrent* updates)

Purpose

Sequential edge delete



Concurrent edge delete



Outline

- Previous works
- Results of the paper
- Complexity measures
- Decremental algorithm
- Incremental algorithm
- Conclusions and future works

Previous Works

- Classical Bellman-Ford and its variations:
  - Complexity: exponential
  - Drawback: Counting Infinity and Looping phenomena
- Awerbuch, Cidon and Kutten, 1990
  - Complexity: $\Theta(n)$ messages and $O(n^2)$ space per node
  - Drawback: is not able to concurrently update shortest paths
- Italiano, 1991
  - Complexity: $O(n \log(nW))$ messages and $O(n)$ space per node
  - Drawback: is not able to concurrently update shortest paths
- Ramarao and Venkatesan, 1992
  - Complexity: $O(n^3)$ messages and $O(n)$ space per node
  - Drawback: is not able to concurrently update shortest paths
- Cicerone et al, 2003
  - Complexity: $O(\text{maxdeg } \Delta_\sigma)$ messages and $O(n)$ space per node
  - Drawback: is not able to concurrently update shortest paths

Results of the paper

By the above analysis, two classes of algorithms are known:

2. Those which are not able to concurrently update shortest paths
3. Those which are able to concurrently update shortest paths but
   - either they suffer of the looping and counting phenomena, or
   - their convergence can be very slow in the case of weight increase operations (possibly infinite)

This paper provides a partially dynamic algorithm that:

- is able to concurrently update shortest paths
- avoids the looping and counting phenomena
- converges fast

Results of the paper

In Detail:

- we propose a new decremental algorithm
  - Complexity: O(maxdeg Δ^2) messages and O(maxdeg Δ) space per node
  - is able to concurrently update shortest paths
- we propose a new incremental algorithm
  - works also in the concurrent case
  - Complexity: O(maxdeg Δ) messages and O(n) space per node

Here Δ is the number of nodes affected by a set of weight change operations

Complexity measures

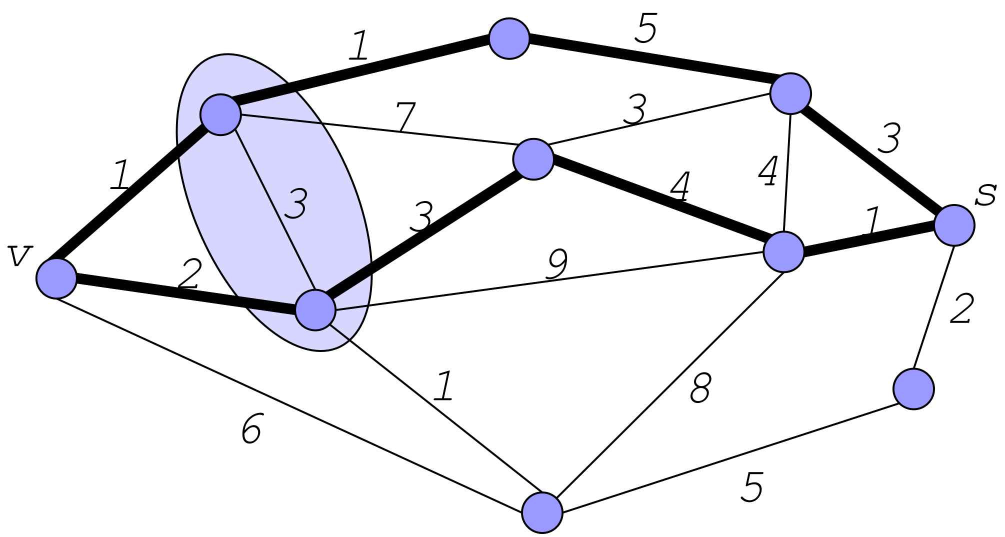
Given a set of k weight changes $\sigma_1, \sigma_2, \ldots, \sigma_k$ and a source node s:

- $\delta_{\sigma_i,s}$: set of nodes that change the shortest paths toward s as a consequence of σ_i
- $\bigcup_{s \in V} \delta_{\sigma_i,s}$: nodes affected by σ_i
- the number of affected nodes is at most:

$$\Delta = \sum_{i=1}^{k} \sum_{s \in V} |\delta_{\sigma_i,s}|$$

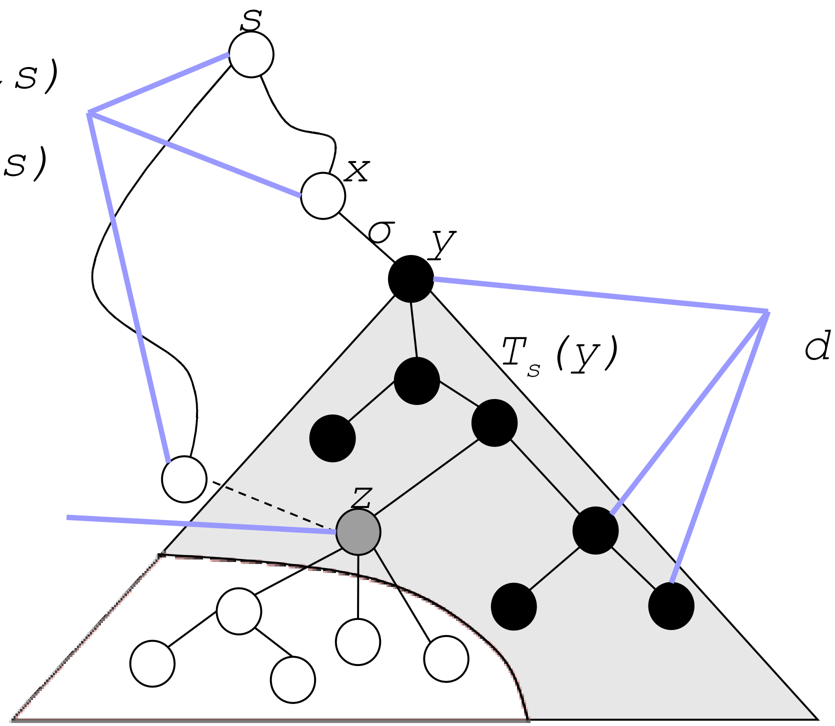
We give the complexity bounds as a function of Δ

Decremental Algorithm - data structures



- There are 2 shortest paths:
  - `v-s` distance, *d(v,s)*: weight of the shortest paths (10)
  - `v-s` via, *via(v,s)*: subset of *N(v)* containing nodes in a shortest path
- Data structures:
  - `d[v,s]`: estimated distance from `v` to `s`
  - `via[v,s]`: estimated via from `v` to `s`

Decremental Algorithm - definitions



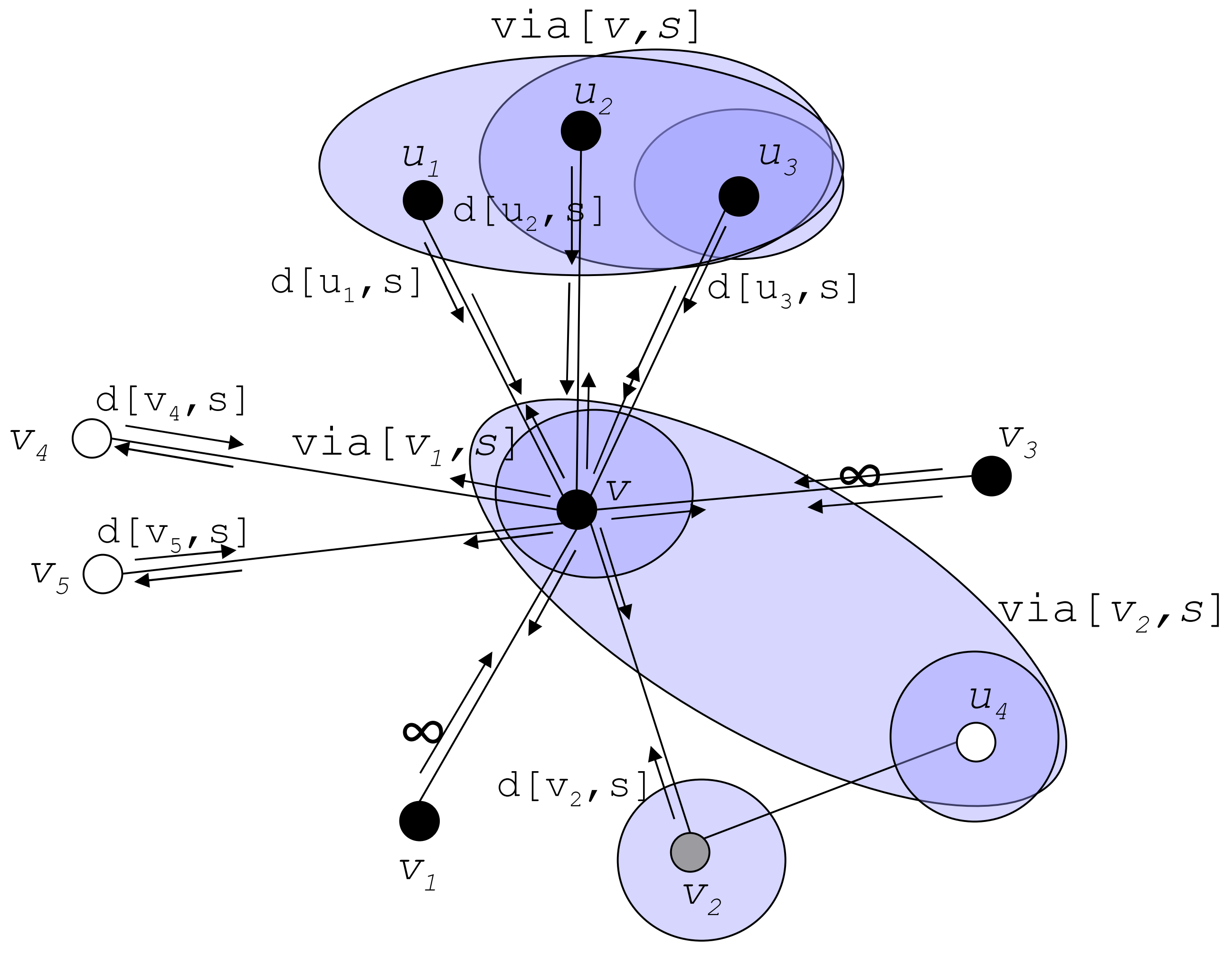
white: $d(v,s)=d^k(v,s)$

$via(v,s) \equiv via^k(v,s)$

black:
$d(v,s) \neq d^k(v,s)$

gray: $d(v,s)=d^k(v,s)$
$via(v,s) \neq via^k(v,s)$

Decremental Algorithm - behavior



Decremental Algorithm – complexity

- Message Complexity
  - Only black nodes send messages
  - For each source `s`, each black node `v` sends `deg(v)` messages at each update
  - There are at most $|\delta_{\sigma_i,s}|$ updates
  - There are $|\delta_{\sigma_i,s}|$ black nodes

$$\sum_{i=1}^{k}\sum_{s\in V}\left(maxdeg\cdot|\delta_{\sigma_i,s}|^2\right)\leq maxdeg\cdot\Delta^2$$

- Space Complexity
  - `via[v,s]` contains at most `deg(v)` elements
  - `via[v,*]` requires `O(deg(v) n)`
  - `d[v,*]` requires `O(n)`
  - At most: `O(maxdeg n)`

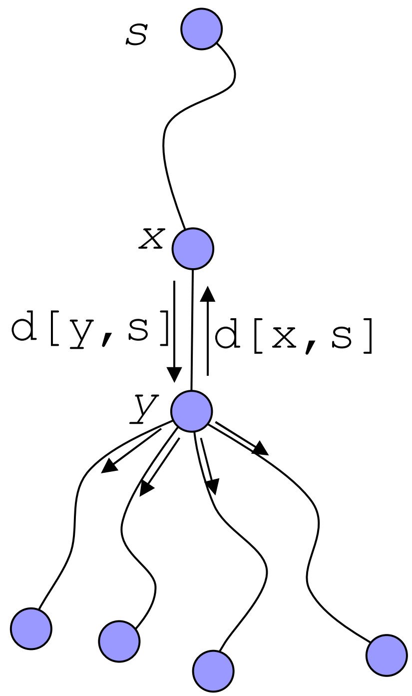
Incremental Algorithm - data structures

The same as the decremental algorithm but:

`via[v,s]` stores <u>only one</u> node in *via(v,s)*

`via[*,s]` induces a shortest paths tree

Incremental Algorithm - behavior



- y is closest than x to s, then y updates its data structures
- Each shortest path that changes contains the edge (x,y)
- y propagates the algorithm in the shortest paths tree induced by `via[*,y]`

Incremental Algorithm – complexity

- Message Complexity
  - For each source s, for each node v, there are at most $|\delta_{\sigma_i,s}|$ updates
  - v sends $deg(v)$ messages at each update

$$\sum_{i=1}^{k} \sum_{s \in V} (maxdeg \cdot |\delta_{\sigma_i,s}|) = maxdeg \cdot \Delta$$

- Space Complexity
  - `via[v,*]` requires $O(n)$
  - `d[v,*]` requires $O(n)$

Conclusions and future works

- We propose a pair of partially dynamic algorithms that are able to concurrently update shortest paths
  - Decremental algorithm complexity: O(maxdeg Δ^2) messages and O(maxdeg Δ) space per node
  - Incremental algorithm complexity: O(maxdeg Δ) messages and O(n) space per node
- Future works:
  - Fully dynamic algorithms
  - Experimental evaluation